Novel Internal Wave paradigm
Strongly Nonlinear Scale Separated Interactions

Yuri V Lvov
Department of Mathematical Sciences,
Rensselaer Polytechnic Institute
Troy NY 12180
phone: (518) 276 68993 fax: (518) 276 4820 email: lvovy@rpi.edu

Award Number: N00014-09-1-0515
http://www.rpi.edu/~lvovy

LONG-TERM GOALS

My long-term goal is to understand and predict spectral energy density of internal waves in the ocean, and the effects of internal waves on oceanic large scale motions.

OBJECTIVES

First-Principle theory of strongly nonlinear wave-wave interactions for internal waves.

APPROACH

Present study is based on the extension of the wave turbulence theory to the internal waves in the ocean. In particular, we attempt to go beyond traditional wave turbulence assumptions of weak nonlinearity and resonant interactions. This study is characterized by strong synergy with the work by Dr. Kurt Polzin (WHOI).

WORK COMPLETED

We have derived an extension of the traditional kinetic equation for internal waves that takes into account not only resonant, but near resonant interactions. This derivation is based on the Dyson-Wild diagrammatic technique. The rational for inclusion of the near-resonant interactions is the following: kinetic equation shows that the amplitude of the waves do tend to change as a result of wave-wave interactions. Thus the life time of the waves is necessarily finite. Consequently waves can not be strictly resonant, as strict resonances appears only for the waves with infinite lifetime. This is related to the Fourier uncertainty principle.

Based on this novel form of the kinetic equation we have evaluated the characteristic times scales of the change of the Garrett and Munk spectrum. First we used traditional Resonant Interactions Approximation (RIA), i.e. the usual kinetic equation that we have derived in References [1-3], The results are presented in Figure ONE. We see that the resulting time scales are of the order of few wave periods – slower for small vertical wavenumber, and faster for large vertical wavenumber. What is most disturbing, is the existence of the region where the wave spectrum evolve faster then the wave

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **2009**	2. REPORT TYPE	3. DATES COVERED **00-00-2009 to 00-00-2009**
4. TITLE AND SUBTITLE **Novel Internal Wave paradigm Strongly Nonlinear Scale Separated Interactions**		5a. CONTRACT NUMBER
		5b. GRANT NUMBER
		5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)		5d. PROJECT NUMBER
		5e. TASK NUMBER
		5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Rensselaer Polytechnic Institute,Department of Mathematical Sciences,Troy,NY,12180**		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR’S ACRONYM(S)
		11. SPONSOR/MONITOR’S REPORT NUMBER(S)
12. DISTRIBUTION/AVAILABILITY STATEMENT **Approved for public release; distribution unlimited**		
13. SUPPLEMENTARY NOTES		
14. ABSTRACT		
15. SUBJECT TERMS		

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**4**	

period. This shows quite unequivocally that (I) traditional kinetic equation approach to internal waves is NOT self c consistent, as nonlinearity is not weak in any sense. (II) Garrett-and-Munk spectrum is not a steady state of the resonant kinetic equation. Note also that the PSI and ID scatterings are seen distinctly on Figure ONE.

We then allowed the waves to be not only resonant, but near-resonant as well. Inclusion of the near-resonant waves is a tricky process since it is necessary to characterize the degree by which one allow the resonant conditions to be not satisfied. In particular we have chosen a simple phenomenological model that described in details in our submitted paper. Resulting characteristic time scales are presented in Figure TWO. We see that the resulting nonlinearity levels are much smaller than in Figure ONE. Therefore it appears that inclusion of the near-resonant waves into account **REDUCES** the level of effective nonlinearity in the system. In other words, inclusion of near-resonant interactions **HEAL** the problematic traditional kinetic equation. Interestingly, the PSI and ID regimes that were seen on the Figure ONE, are much less pronounced on this figure.

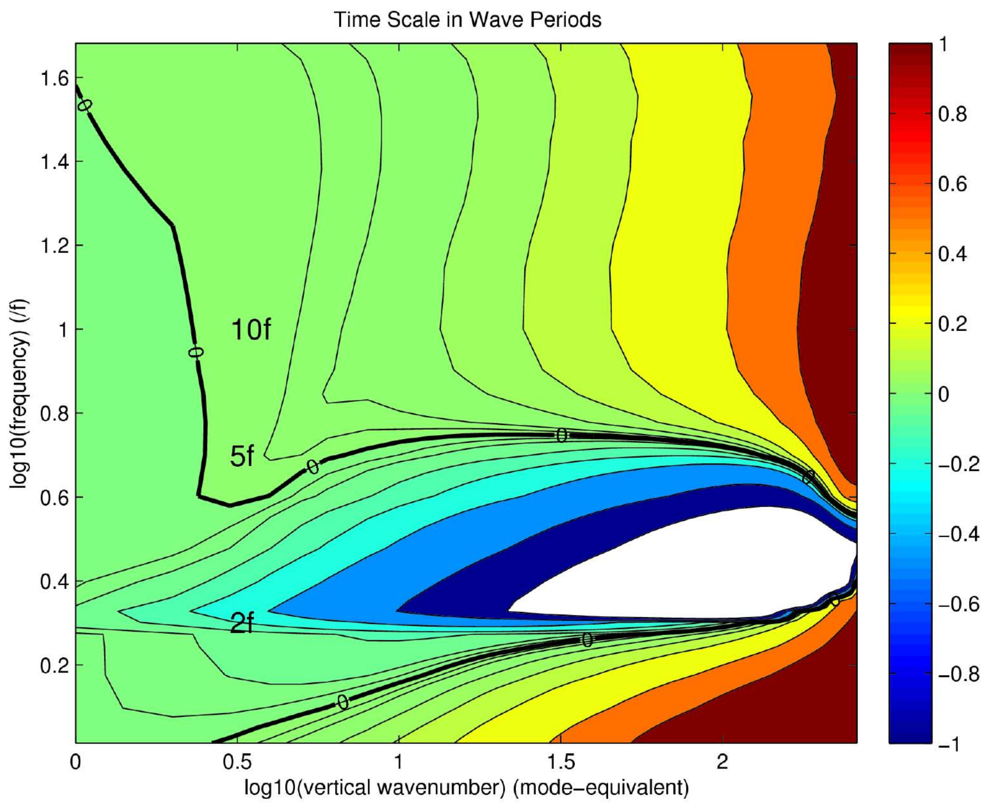

Figure ONE Characteristic time scales of the GM spectrum in Resonant Interactions Approximation (RIA)

Characteristic time scales of the evolution of the Garrett and Munk spectrum calculated by the (almost) resonant kinetic equation. Time is measured in wave periods. Time evolution for large wave numbers is faster than a wave period designated by white color on the plot. This plot shows that GM is not a steady state of the kinetic equation, and that it is not self-consistent approach to use kinetic equation for internal waves in the ocean.

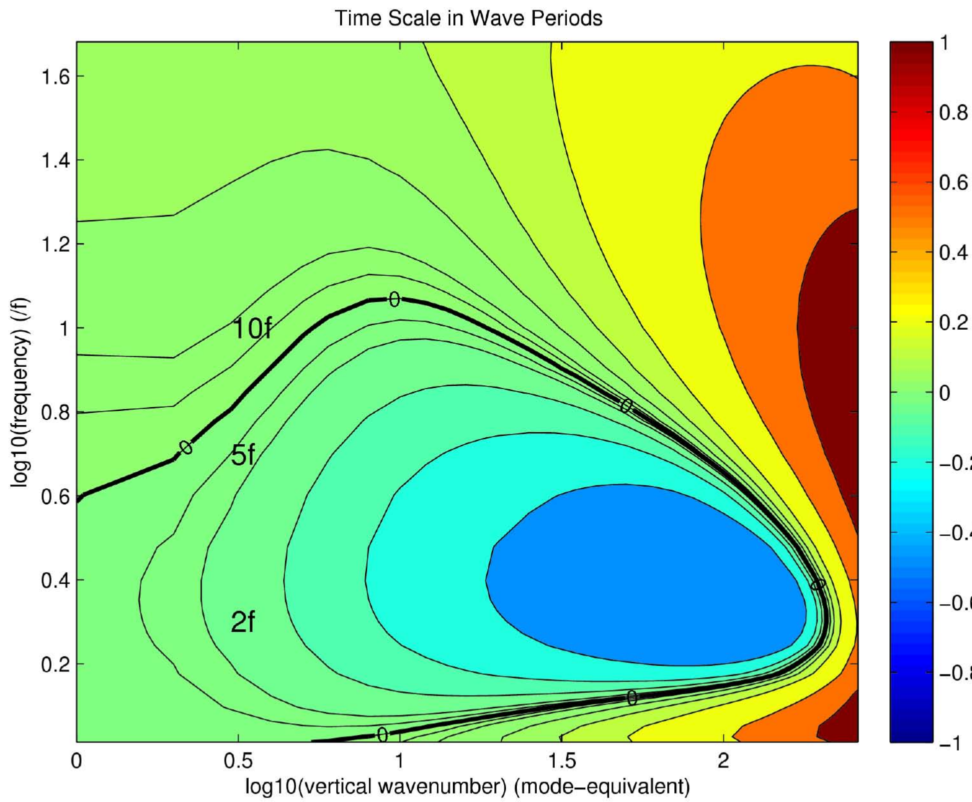

Figure TWO Characteristic time scales of the GM spectrum with near-Resonant Interactions included

Characteristic time scales of the evolution of the Garrett and Munk spectrum calculated with resonant and near-resonant interactions included. Figure shows that these characteristic time scales are **less** then the wave period, and is much less for small vertical wavenumber. Therefore the inclusion of the near-resonant terms makes approach of wave turbulence more self-consistent.

RESULTS

We have shown that the Garrett and Munk spectrum of the internal waves in the ocean is NOT a steady state solution of the traditional kinetic equation. This statement is at odds with traditional wisdom about wave-wave interactions in the ocean.

Boltzmann rate for the Garrett and Munk spectrum is too large for the system to be considered weakly nonlienar.

Inclusion of the near-resonant interactions makes a system of internal waves less nonlinear (more weakly nonlinear). This will lead to more self-consistent approach to wave-wave interactions.

IMPACT/APPLICATIONS

Successful completion of this effort will lead to effective and simple internal waves parametrization schemes used for global ocean modeling.

RELATED PROJECTS

REFERENCES

1] Lvov, Y.~V., and E. G. Tabak, 2001: Hamiltonian formalism and the Garrett and Munk spectrum of internal waves in the ocean. Phys. Rev. Lett., 87, 169501.

2] Lvov, Y.~V., and Tabak E.~G., 2004: A Hamiltonian Formulation for Long Internal Waves. Physica D195 106--122.

3] Lvov, Y.~V., Polzin K.~L. and Tabak E., 2004: Energy spectra of the ocean's internal wave field: theory and observations. Phys. Rev. Lett. 92}, 128501.

PUBLICATIONS

Yuri V Lvov, Kurt L Polzin, Naoto Yokoyama, "Wave-wave interactions in stratified fluids: A comparison between different approaches", Journal of Physical Oceanography, Submitted to JPO.